



Quiz 1, Date: 7 April, 2024
Instructor: Ankan Kar

Timing: 2:00 PM to 4:30 PM

Score rule as per IMO

All questions carry equal marks, try as much as possible

1. The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2+i) = 0$. What is $a + b + c + d$?
2. The equation $z^6 + z^3 + 1 = 0$ has complex roots with argument θ between 90° and 180° in the complex plane. Determine the degree measure of θ .
3. Let $P(x)$ be a non-zero polynomial with integer coefficients. If $P(n)$ is divisible by n for each positive integer n , what is the value of $P(0)$?
4. Consider the sequence $1, 3, 3, 3, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7, \dots$ and evaluate its 2016th term.
5. Let a and b be natural numbers such that $2a - b$, $a - 2b$, and $a + b$ are all distinct squares. What is the smallest possible value of b ?
6. Let p be a prime number greater than 3. Prove that $p^2 - 1$ is divisible by 24.

End